

**Practice Paper – Set 2**

**A Level Mathematics A  
H240/01 Pure Mathematics**

**MARK SCHEME**

**Duration: 2 hours**

**MAXIMUM MARK    100**

**Final**

## Text Instructions

## 1. Annotations and abbreviations

Annotation in scoris	Meaning
✓ and ✕	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	Meaning
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This question included the instruction: In this question you must show detailed reasoning.

## 2. Subject-specific Marking Instructions for A Level Mathematics A

- a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.
- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.
- c The following types of marks are available.

### M

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

### A

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

### B

Mark for a correct result or statement independent of Method marks.

### E

Mark for explaining a result or establishing a given result. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep\*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.  
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.
- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for  $g$ . E marks will be lost except when results agree to the accuracy required in the question.
- g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.
- h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.
- i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		Answer	Marks	AO	Guidance	
1	(i)	$(x \pm 3)^2 + (y \pm 2)^2 \dots$ $(-3, 2)$	M1 A1 [2]	1.1a 1.1	Attempt to complete the square State correct centre www	Ignore constant term(s)
1	(ii)	$13 + k = 16$ $k = 3$	M1 A1 [2]	1.1a 1.1	Attempt to link 9, 4, 16 and $k$ Obtain $k = 3$	
2	(i)	$n = \pm 5$ $ 2(-5) - 3  = 13 \quad  2(5) - 3  = 7$  13, as $13 > 7$	B1 M1  A1 [3]	1.2 2.1  2.4	Identify that $n = \pm 5$ Substitute both $n = \pm 5$ into expression Conclude with 13 and explanation	
2	(ii)	$3x - 6 = x - 6$ , so $x = 0$  $3x - 6 = -x + 6$ $4x = 12$ $x = 3$	B1  M1  A1 [3]	1.1  1.1a  1.1	Obtain $x = 0$  Attempt to solve equation, with $3x$ and $x$ of opposite signs Obtain $x = 3$	OR M1 square both sides A1 Obtain $x = 0$  A1 Obtain $x = 3$
3		$b^2 - 4ac = (k - 6)^2 - 8k$ $k^2 - 20k + 36$ $k = 2, k = 18$  $k < 2, k > 18$  $(\infty, 0) \cup (0, 2) \cup (18, \infty)$	M1 A1 M1  M1  A2  [6]	1.1a 1.1a 1.2  1.1a  2.3  2.5	Attempt discriminant Obtain correct 3 term expression Find critical values for $k$ , eg by attempting to solve discriminant $> 0$ Chooses 'outside region' of their inequality A1 for stating $k \neq 0$ OR A1 for $(\infty, 2) \cup (18, \infty)$	Allow any sign, including =  Allow 'or' but not 'and'  Any correct set notation

Question		Answer	Marks	AO	Guidance
4	(i)	Sketch of $y = \frac{3}{x^2}$ Sketch of $y = x^2 - 2$ Intercepts of (0, -2), ( $\sqrt{2}$ , 0), ( $-\sqrt{2}$ , 0)	<b>B1</b> <b>M1</b> <b>A1</b> <b>[3]</b>	<b>1.1</b> <b>1.1</b> <b>1.1</b>	Must be in both quadrants, with axes intended as asymptotes Positive quadratic, symmetrical in y-axis All 3 intercepts correct Allow just eg $\sqrt{2}$ if marked on relevant axis
4	(ii)	<b>DR</b> $3 = x^4 - 2x^2$ $(x^2 - 3)(x^2 + 1) = 0$ $x^2 = 3$ $x = \pm\sqrt{3}$ points are ( $\sqrt{3}$ , 1) and ( $-\sqrt{3}$ , 1)  $x^2 = -1$ has no solutions as $x^2 \geq 0$	<b>M1</b> <b>M1</b> <b>M1</b> <b>A1</b> <b>A1</b>  <b>B1</b> <b>[6]</b>	<b>3.1a</b> <b>1.1a</b> <b>1.1a</b> <b>1.1</b> <b>2.2a</b>  <b>2.3</b>	Equate and remove fractions Attempt to solve disguised quadratic Attempt to find $x$ from their roots Correct $x$ values Both correct coordinates  Justify no roots for $x^2 = -1$
5	(i)	$u_{10} = 150 + 9 \times 16$ $= 294$ ice creams profit = $294 \times \text{£}1.25 = \text{£}367.50$	<b>B1</b> <b>M1</b> <b>A1FT</b> <b>[3]</b>	<b>3.1b</b> <b>1.1</b> <b>3.2a</b>	Identify AP, with $a = 150$ and $d = 16$ Correct $u_{10}$ Correct profit for their $u_{10}$ Units required
5	(ii)	$\text{£}5000 \div \text{£}1.25 = 4000$ $S_N = 0.5N(300 + (N - 1)16)$  $150N + 8N(N - 1) > 4000$ $8N^2 + 142N - 4000 > 0$  $N = 15.18$ (and possibly -32.9) Week 16	<b>B1</b> <b>M1</b>  <b>A1</b>  <b>M1</b> <b>A1</b> <b>[5]</b>	<b>3.1b</b> <b>3.4</b>  <b>3.1a</b>  <b>1.1</b> <b>3.2a</b>	Identify that 4000 sales are reqd Attempt $S_N$ of AP, with $a = 150$ and $d = 16$  Link to 4000 (any sign) and rearrange to 3 term quadratic  Attempt to solve quadratic Conclude with Week 16 only Or use $d = \text{£}20$ Or $d = 20$  Or link to 5000 (any sign) and rearrange to 3 term quadratic <b>BC</b> Allow 'during Week 16'

Question		Answer	Marks	AO	Guidance	
5	(iii)	Sales cannot continue to increase for ever Weekly sales could fluctuate depending on the weather	E1 E1 [2]	3.5b 3.5b	Refer to trend not continuing Refer to changes week by week	Any two different reasons
6		Assume that $\sqrt{7}$ is rational ie $\sqrt{7} = \frac{a}{b}$ , where $a$ and $b$ have no common factors $b\sqrt{7} = a$ , so $7b^2 = a^2$ So $a^2$ must be a multiple of 7, which means that $a$ is a multiple of 7 as well, so $a = 7k$ $7b^2 = (7k)^2 \Rightarrow 7b^2 = 49k^2 \Rightarrow b^2 = 7k^2$ This implies that $b$ is a multiple of 7, but it was assumed at start that $a$ and $b$ had no common factors, so this contradicts initial assumption. Hence $\sqrt{7}$ cannot be written as $\frac{a}{b}$ so it is irrational.	E1 M1 E1 M1 E1 [5]	2.1 2.2a 2.4 2.2a 2.4	Proof must start with an assumption for contradiction Rearrange and square both sides Identify that $a = 7k$ Substitute $a = 7k$ and simplify Conclude appropriately	Condone not stating that $a$ and $b$ have no common factors  Must have stated at start that $a$ and $b$ have no common factors
7	(i)	$\frac{\sin\theta}{4.8} = \frac{\sin 25^\circ}{2.2}$ $\theta_1 = 67.2^\circ$ , so bearing is $067^\circ$ $\theta_2 = 180^\circ - 67^\circ$ $\theta_2 = 112.8^\circ$	M1 A1 M1 A1 [4]	3.1a 1.1 3.1a 1.1	Attempt correct use of the sine rule to find at least one angle Obtain $067^\circ$ , or better Attempt correct method for $\theta_2$ Obtain $113^\circ$ , or better	Must use $25^\circ$  3 figure bearing required $180^\circ$ – their angle
7	(ii)	$d = 4.8\sin 25^\circ$ $= 2.03$ km	M1 A1 [2]	3.1a 1.1	Attempt perpendicular distance Obtain 2.03 km, or better	Allow alt complete methods

Question		Answer	Marks	AO	Guidance	
7	(iii)	Coastline may not be straight between $P$ and $Q$	<b>E1</b>  <b>[1]</b>	<b>3.5b</b>	Any sensible reason	Stations may not be exactly on coastline
8	(i)	$\sec x = (\cos x)^{-1}$ $\frac{dy}{dx} = -(\cos x)^{-2}(-\sin x)$  $\frac{dy}{dx} = \frac{1 \times \sin x}{\cos x \times \cos x} = \sec x \tan x$ <b>A.G.</b>	<b>B1</b> <b>M1</b>  <b>A1</b> <b>[3]</b>	<b>2.1</b> <b>2.1</b>  <b>2.4</b>	Correct definition for $\sec x$ so i Attempt chain rule or quotient rule  Show given answer	At least one step needed
8	(ii)	<b>DR</b> $\sec^2 2x + 2\sec 2x \tan 2x + \tan^2 2x$ $2\sec^2 2x + 2\sec 2x \tan 2x - 1$ $\int f(x) dx = \tan 2x + \sec 2x - x$  $(\sqrt{3} + 2 - \frac{1}{6}\pi) - (\frac{1}{3}\sqrt{3} + \frac{2}{3}\sqrt{3} - \frac{1}{12}\pi)$ $2 - \frac{1}{12}\pi$	<b>B1</b> <b>M1</b> <b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b> <b>[6]</b>	<b>3.1a</b> <b>1.2</b> <b>1.1a</b> <b>1.1</b> <b>1.1a</b> <b>1.1</b>	Correct expansion of bracket Use $\tan^2 2x = \sec^2 2x - 1$ Attempt integration Correct integral Attempt use of limits Obtain $2 - \frac{1}{12}\pi$	One trig term correct  Correct order and subtraction
9	(i)	$\frac{A}{(1-2x)} + \frac{B}{(2+x)} + \frac{C}{(2+x)^2}$ $A(2+x)^2 + B(1-2x)(2+x) + C(1-2x)$ $= 5 + 4x - 3x^2$  $A = 1$ $B = 2$ $C = -3$	<b>M1</b>  <b>M1</b>  <b>A1</b> <b>A1</b> <b>A1</b> <b>[5]</b>	<b>2.1</b>  <b>1.1a</b>  <b>1.1</b> <b>1.1</b> <b>1.1</b>	Correct partial fractions  Correct equation  Correct value for $A$ Correct value for $B$ Correct value for $C$	

Question			Answer	Marks	AO	Guidance	
9	(ii)		$1 + 2x + 4x^2$	<b>B1</b>	<b>1.1a</b>	Correct expansion of $(1 - 2x)^{-1}$	
			$0.5(1 + 0.5x)^{-1}$ or $0.25(1 + 0.5x)^{-2}$	<b>B1</b>	<b>3.1a</b>	At least one correct expression	
			$1 - 0.5x + 0.25x^2$	<b>M1</b>	<b>1.1</b>	Attempt expansion of $(1 + 0.5x)^{-1}$	
			$1 - x + 0.75x^2$	<b>M1</b>	<b>1.1</b>	Attempt expansion of $(1 + 0.5x)^{-2}$	
			$\frac{5}{4} + \frac{9}{4}x + \frac{59}{16}x^2$	<b>A1FT</b>	<b>1.1a</b>	Correct expansion, following their A, B and C	Any equiv, including decimals
				<b>[5]</b>			
9	(iii)		$ x  < 0.5$	<b>B1</b> <b>[1]</b>	<b>2.5</b>	Any equivalent notation	
10			<b>DR</b> $2x - 4y - 4x \frac{dy}{dx} + 24y^2 \frac{dy}{dx} = 0$  $\frac{dy}{dx} = 0$  $2x - 4y = 0$ $x^2 - 2x^2 - x^3 + 4 = 0$ $x^3 - x^2 - 4 = 0$ $f(2) = 0$  $(x - 2)(x^2 + x + 2) = 0$  $\Delta = -7 < 0$ so quadratic has no real roots, hence just one stationary point	<b>M1*</b>  <b>A1</b> <b>M1d*</b>  <b>A1</b> <b>M1</b> <b>A1</b> <b>B1</b>  <b>M1</b>  <b>A1</b> <b>E1</b>  <b>[10]</b>	<b>1.1a</b>  <b>1.1</b> <b>1.1a</b>  <b>1.1</b> <b>1.1a</b> <b>1.1</b> <b>3.1a</b>  <b>2.1</b>  <b>2.1</b> <b>2.4</b>	Attempt implicit differentiation  Obtain correct derivative Either rearrange and use, or substitute Obtain $2x - 4y = 0$ , or equiv Eliminate $x$ or $y$ from eqn of curve Obtain correct cubic Identify $x = 2$ as root or $(x - 2)$ as factor Attempt to factorise cubic – any valid method  Correct quadratic quotient Justify one stationary point	Deal with at least one $y$ term correctly  <b>OR</b> $4y^2 - 8y^2 + 8y^3 - 4 = 0$ <b>OR</b> $2y^3 - y^2 - 1 = 0$ <b>BC</b> <b>OR</b> $f(1) = 0$ <b>OR</b> $(y - 1)(2y^2 + y + 1) = 0$ Allow for dividing by root of their cubic  Correct working only
11	(i)	(a)	2.5m	<b>B1</b> <b>[1]</b>	<b>3.4</b>	State 2.5m	

Question			Answer	Marks	AO	Guidance
11	(i)	(b)	$30t = 90$ $t = 3$ , hence time is 0300	<b>M1</b> <b>A1</b> <b>[2]</b>	<b>1.1</b> <b>2.2b</b>	Identify that $30t = 90$ so Solve for $t$ , and deduce time is 0300
11	(ii)		$1.7 + 0.8\sin(30t) = 1.2$ $\sin(30t) = -0.625$ $30t = 218.68$ $t = 7.289$  hence time is 0717	<b>M1</b>  <b>M1</b>  <b>A1</b> <b>A1FT</b>  <b>[4]</b>	<b>3.4</b>  <b>1.1a</b>  <b>1.1</b> <b>2.2a</b>	Equate to 1.3 and rearrange to $\sin(30t) = k$ Attempt to find value for $t$ from angle in third quadrant Obtain 7.29, or better Obtain time as 0717  FT their decimal value for $t$ , as long as 2dp or better
11	(iii)		$a = 1.9$ , $b = 1.2$ $c = 30$ $1.9 + 1.2\sin(150 + d) = 3.1$ $150 + d = 90$ $d = -60$	<b>B1</b> <b>B1</b> <b>M1</b>  <b>A1</b> <b>[4]</b>	<b>3.3</b> <b>3.3</b> <b>3.1b</b>  <b>1.1</b>	Correct values for $a$ and $b$ Correct value for $c$ Attempt to find $d$ eg use $f(2) = 3.1$  Obtain correct value for $d$
11	(iv)		Identify decrease with reason	<b>B1</b>   <b>[1]</b>	<b>3.5c</b>	Eg There is a longer time between maximum heights, so the value of $c$ will need decrease to give longer period for the sine model

Question		Answer	Marks	AO	Guidance
12	(i)	$2udu = dx$ $\int 2ue^u du$  $2ue^u - \int 2e^u du$ $2ue^u - 2e^u$ $2\sqrt{x+1}e^{\sqrt{x+1}} - 2e^{\sqrt{x+1}} + c$	<b>B1</b> <b>M1</b> <b>A1</b>  <b>M1</b> <b>A1</b> <b>A1</b> <b>[6]</b>	<b>1.1</b> <b>3.1a</b> <b>2.4</b>  <b>1.1a</b> <b>1.1</b> <b>1.1</b>	Correct relationship so Convert to integrand in terms of $u$ Fully correct, including $du$  Attempt integration by parts  Fully correct in terms of $u$ Fully correct in terms of $x$  A0 if $du$ never seen, but all remaining marks available  Condone no $+c$ $+c$ now required
12	(ii)	$x = (\ln y)^2 - 1$	<b>B1</b> <b>[1]</b>	<b>2.1</b>	Correct equation in form $x = f(y)$
12	(iii)	the equation in (ii) gives the area between curve and $y$ -axis $y = e^4 \Rightarrow x = 15, \quad y = e \Rightarrow x = 0$ area between curve and $x$ -axis is $(8e^4 - 2e^4) - (2e - 2e) = 6e^4$ area of rectangle is $15e^4$ hence reqd area is $15e^4 - 6e^4 = 9e^4$ <b>A.G.</b>	<b>B1</b> <b>B1</b> <b>M1</b> <b>A1</b> <b>[4]</b>	<b>2.4</b> <b>2.1</b> <b>2.2a</b> <b>2.4</b>	Identify geometrical relationship  Identify $x$ limits Use $x$ limits in integral from (i)  Conclude convincingly

H240/01

Mark Scheme

Practice Paper (March 2019)

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